MODELING WITH EXPONENTIAL FUNCTIONS

A.9E **The student is expected to write, using technology, exponential functions that provide a reasonable fit to data and make predictions for real-world problems.**

TELL ME MORE...

$$f(x) = ab^x$$

Exponential functions are based on a constant multiplier, or base, that is multiplied by itself x times. Repeated multiplication is represented with an exponent. Hence, the independent variable in an exponential function appears in the exponent of the function. You can write an exponential function in general form, $f(x) = ab^x$, where a and b are real numbers, b represents the constant multiplier, or base, and a represents an initial value or starting point.

Modeling with Transformations

Make a scatterplot of the data. Beginning with the parent function $f(x) = 2^x$, adjust the starting point, a, to match your data. Then, adjust the base, b, so that the curve aligns with the data points.

Modeling with Regression

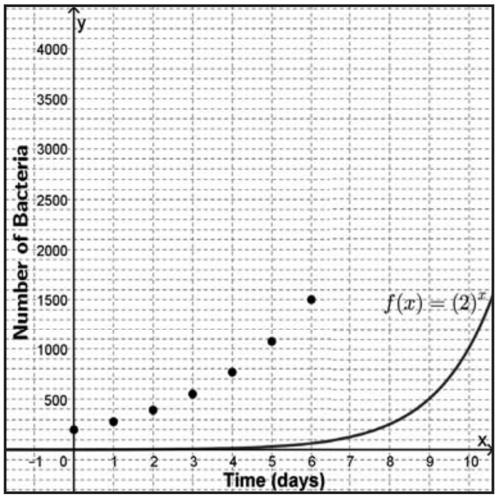
Enter the data into a spreadsheet, graphing technology, or app. Use the technology's regression programming to generate a curve of best fit. Graph the curve on top of a scatterplot of your data to make sure the curve fits.

EXAMPLES

EXAMPLE 1: LaQuan is a medical technologist studying bacteria cultures. For one particular bacteria strain, he started with 200 bacteria in a Petri dish. The table shows the growth of the bacteria culture over time. Based on the data, write a function that best models the data and use the function to predict when the bacteria culture will reach 4000 bacteria.

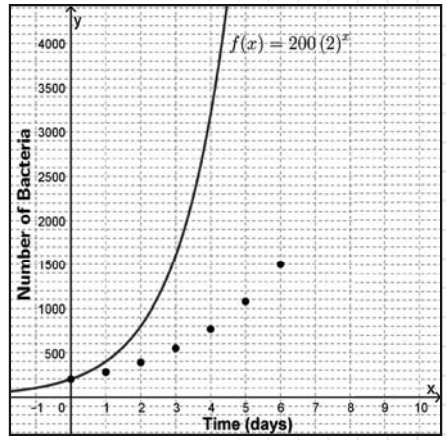
Time (days), x	Number of Bacteria, $p(x)$
0	200
1	280
2	390
3	550
4	768
5	1075
6	1500

STEP 1 Make a scatterplot of the data. Graph the exponential parent function, $f(x) = 2^x$, over the scatterplot.



STEP 2 Transform the value of a, the starting point, so that it aligns with (0, 200), the starting point from the table.

- In the general form $f(x) = ab^x$, let a equal the value of the starting point, or $a = 200$.
- Graph $f(x) = 200(2)^x$

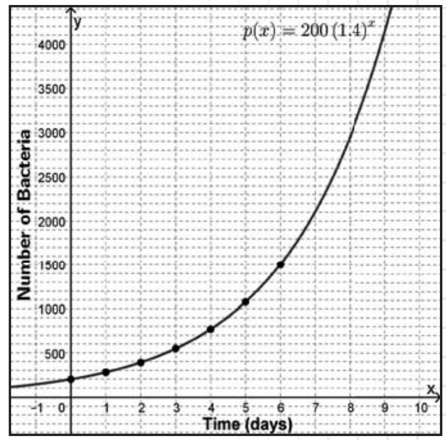


STEP 3 Transform the curvature of the graph by adjusting the base, b.

- Use the data in the table to estimate a value of b. Look for a constant ratio between successive function values.

Time (days), x	Number of Bacteria, $p(x)$
0	200
1	280
2	390
3	550
4	768
5	1075
6	1500

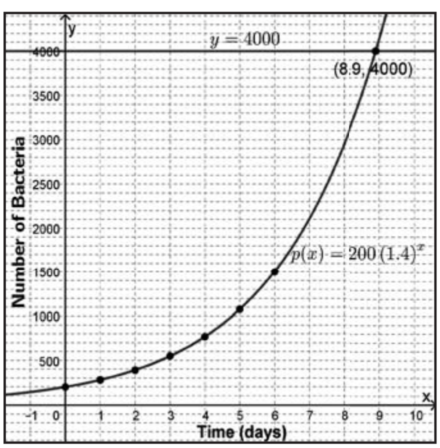
$280 \div 200 = 1.4$
$390 \div 280 \approx 1.39$
$550 \div 390 \approx 1.41$
$768 \div 550 \approx 1.40$



- For the first few rows of data, you multiply the previous function value by approximately 1.4 to generate the next function value. Graph $p(x) = 200(1.4)^x$.

p*(*x*) = 200(1.4)*ˣ

STEP 4 The number of bacteria is the dependent variable. Let $p(x) = 4000$ and graphically estimate the solution to the equation $4000 = 200(1.4)^x$.



The number of bacteria will reach 4000 in about 8.9 days.

YOU TRY IT!

Write a function that models the data shown.

x	y
0	75
1	262
2	920
3	3200
4	11,225

Value of a: ___________ b: ___________

Function: ______________________

EXAMPLE 2: A company that manufactures car parts found a new process that allows them to reduce the cost to manufacture a certain part. The table shows the cost per part for the first few months of the process. Write $c(x)$ that describes the cost as a function of time.

Month, x	Cost per Part, $c(x)$
0	600
1	480
2	380
3	310
4	250
5	200
6	155
7	125
8	100

STEP 1 Use regression with technology to determine an appropriate function model. Enter the data into your technology.

L1	L2	L3	L4	L5
0	600	------	------	------
1	480			
2	380			
3	310			
4	250			
5	200			
6	155			
7	125			
8	100			
------	------			

STEP 2 Use the technology's exponential regression algorithm to generate an exponential regression equation.

$c(x) = 602.197(0.799)^x$

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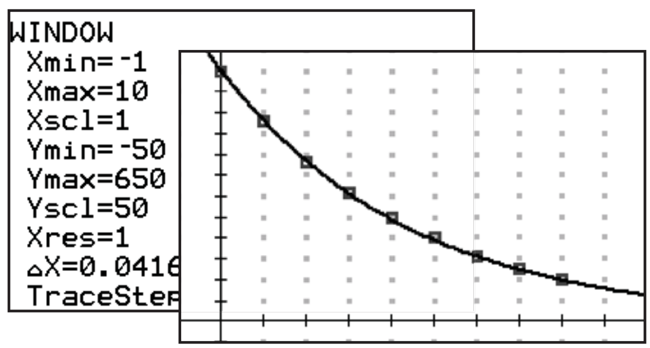
ExpReg
y=a*b^x
a=602.197417
b=0.7993790531
r²=0.9996543862
r=-0.9998271782
```

STEP 3 Graph the exponential regression equation for $c(x)$ over a scatterplot of the data. Compare the graph of the function model with the scatterplot of the actual data.

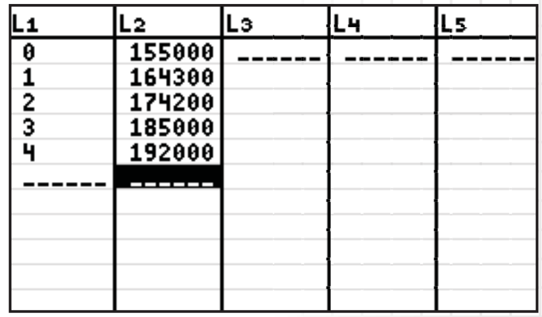
The graph seems to pass through or very near every data point in the scatterplot. The graph follows the general trend of the data.



EXAMPLE 3: The median sale price of a single family home in a city in 2010 was \$155,000. The table shows how that value changed over the next several years. If prices continue changing at this rate, write an exponential function, $f(x)$, that best describes the data. What will be the median sale price of a single family home in 2020?

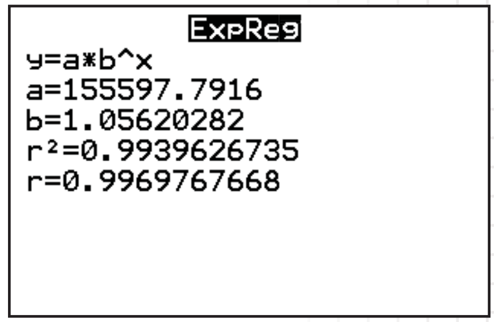
Year since 2010, x	0	1	2	3	4
Sale Price, $f(x)$	\$155,000	\$164,300	\$174,200	\$185,000	\$192,000

STEP 1 Use regression with technology to determine an appropriate function model. Enter the data into your technology.



STEP 2 Use the technology's exponential regression algorithm to generate an exponential regression equation.

***f*(*x*) = 155597.79(1.056)^x**



STEP 3 Determine the function value for 2020. In this case, the input value (independent variable) is the year since 2010, x, which is 2020 – 2010 = 10. Substitute $x = 10$ into $f(x)$ and evaluate the function.

$$f(x) = 155597.79(1.056)^x$$
$$f(10) = 155597.79(1.056)^{10}$$
$$f(10) \approx 155597.79(1.7244)$$
$$f(10) \approx 268313$$

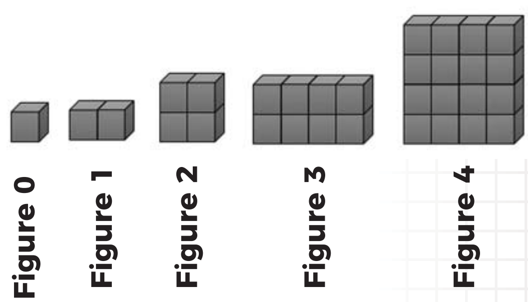
The median sale price in 2020 will be approximately $268,313.

PRACTICE

1. A table of values for the exponential function g is shown. What is the function that is best represented by the data in the table?

x	$g(x)$
0	250
1	125
2	62.5
3	31.25
4	15.625

2. The first 5 figures of a pattern are shown below. Each figure is made up of identical cubes. If the pattern continues, what expression can be used to find the number of cubes that make up Figure n?

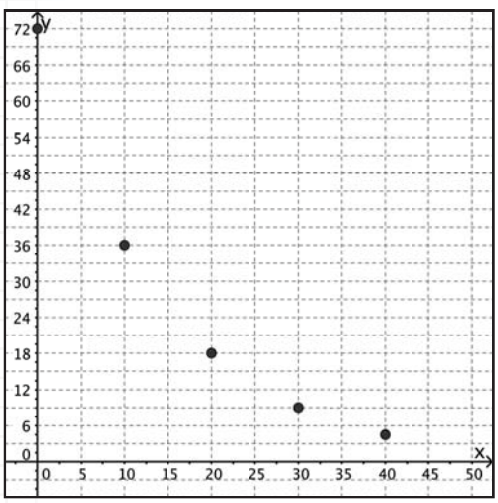


3. Araceli's grandmother opened a savings account for her 10 years ago. The account carries an annual interest rate. The table below shows the balance in the account for some of the years since it was opened.

Year, x	Balance (dollars), y
1	\$1,590.00
3	\$1,786.50
6	\$2,217.80
8	\$2,390.80
10	\$2,686.30

Based on the data, to the nearest percent, what is the approximate interest rate?

4. The graph shows data for the mass of a sample of a radioactive substance based on the time since it began to decay.



Based on the data, which of the following is closest to the approximate time of decay after which the substance will have a mass of 1 gram?

A 50 hours

B 62 hours

C 67 hours

D 100 hours

5. The table shows the temperature of a cup of coffee at various times after being poured into a cup and left to cool.

Time (minutes), x	Temperature (°F), $f(x)$
0	179.5
5	168.7
10	150.3
15	141.7
20	130.0

Based on the data in the table, after about how many minutes will the approximate temperature of the cup of coffee reach 100°F?

6. Carla's pet puppy received an injection. The table shows the amount of medication, M, remaining in the puppy's bloodstream over a period of time, h.

Time (hr)	Medication (mg)
0	275
1	220
2	176
3	140.8
4	112.64
5	90.112

Based on the information in the table, what function rule best models the relationship between the number of hours and the number of milligrams of the medication remaining in the puppy's bloodstream?

F $M = 0.8^h + 275$

G $M = 0.8 \bullet 275^h$

H $M = 0.8h + 275$

J $M = 275 \bullet 0.8^h$

